3401. (a) Take the bear's position as $\mathbf{r}_b = (5t + c)\mathbf{i}$, and find c such that the bear arrives at the correct time.
(b) Use Pythagoras.
(c) Calculate $\frac{dS}{dt}$, and look for SPs.

3402. To visualise this, place a and b on a number line. The odd and even cases are different.

3403. The next move for $\times$ to play is in a corner which is not adjacent to $\bigcirc$. There is only one response to this. Continue in this vein until $\times$ wins.

3404. Let $u = 1 + \ln x$. Note that du appears directly in the integral.

3405. (a) Use $\cos 2\theta = 1 - 2\sin^2\theta$.
(b) Consider the symmetry of the cosine function.
(c) Consider the periodicity of the cosine function.
(d) The lines in (c) form infinitely many squares of the same size.

3406. Firstly, take out a factor of x^2. Then consider the remaining factor as a quadratic in x^4.

3407. (a) Differentiate implicitly with respect to y.
(b) Factorise the LHS as a difference of two squares, then consider dividing by either factor.
(c) Sketch the asymptotes first, then consider any axis intercepts.

3408. Two of the numbers must be positive.

3409. Consider that the shortest distance between two curves must lie along the normal to both.

3410. Expand both top and bottom binomially, noting that only the first two terms of each expansion are relevant.

3411. (a) Find the first, second and third derivatives.
(b) Consider the long-term behaviour of each term individually.

3412. Draw a clear large-scale diagram. Consider the $(2, 2, 1)$ triangle formed by the centres and a point of intersection. Find the angles in this triangle, in exact form. Then calculate the area using sectors, segments and triangles.

3413. (a) Use $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.
(b) Consider $\mathbb{P}(A \cup B) - \mathbb{P}(A \cap B)$.

3414. Consider the equation $x^2 + 4x + 7 = x$.

3415. Start withe RHS. Consider $\cos 3x \equiv \cos(2x + x)$, and use a compound-angle formula, then a double-angle formulae.

3416. Rearrange by putting the x's and y's on opposite sides of the equation, using a log rule, and then exponentiating.

3417. Set up a definite integral with respect to y.

3418. Out of the eight original outcomes, restrict the possibility space using the condition given.

3419. Two are false; one is true. Consider e.g. $(2, 4)$.

3420. The line $y = x + k$ is parallel to the line $y = x$. Hence, reflection in $y = x + k$ can be thought of as reflection in $y = x$ followed by translation by the vector $-k\mathbf{i} + k\mathbf{j}$.

3421. Consider the effect of reflection in the line $x = a$ on the gradients at $x = a$.

3422. Let $u^2 = x - 1$. Differentiate this implicitly and rearrange for dx. Enact the substitution. Rewrite the resulting integrand in partial fractions.

3423. (a) Draw a clear force diagram of the situation, splitting contact forces into horizontal (each hand separate) and vertical (hands combined) components.
Set up equations for horizontal and vertical equilibrium, and for moments around e.g. the centre of mass.
(b) Consider what would happen if you separated the vertical forces exerted by each hand.

3424. Show that g is a polynomial with odd degree.

3425. A "right-circular" cone just means a regular cone. Draw a diagram of the cross-section. Then use the formula πrl for the curved surface area, expressing both r and l in terms of h and θ.

3426. Combine with a log rule, and then express each of the binomial coefficients in factorials.

3427. Write the function out longhand and simplify. Then complete the square in the resulting fraction.

3428. Prove this by direct argument. In this case, the direct argument is construction. A terminating decimal can be expressed as $\frac{a}{10^n}$, where $a, n \in \mathbb{N}$.

3429. (a) Differentiate implicitly with respect to y.
(b) Make x the subject.

3430. Differentiate to find **v**. Then, using Pythagoras, set the square of the magnitude of the velocity to 4. Use a trig identity to simplify and solve.

3431. Show that the boundary equation $\text{f}''(x) = 0$ must have two distinct roots. Interpret this in terms of the degree of the polynomial f''. Integrate twice.

3432. The boundary equations are a pair of intersecting lines and a circle.

3433. Use a double-angle formula for $\tan 2a$. Multiply both sides by the denominator and also by $\tan a$. Solve the quadratic in $\tan a$ to get four roots.

3434. Equate ratios in the GP and differences in the AP. Solve the resulting pair of simultaneous equations.

3435. (a) Draw a force diagram for the relevant box.
   (b) First consider the whole stack as one object.
   (c) Consider the box most likely to buckle, using your answers to (a) and (b).

3436. The question is, having taken out x, whether the remaining quadratic in x^2 has real roots.

3437. Consider what you would need to replace y with in the first graph to get the second graph.

3438. Write out the product rule for $(uv)'$, and integrate both sides of the equation.

3439. Set up an equation of motion along the string, and calculate the mutual acceleration. Hence, find the relative acceleration, and from that the relative position.

3440. Rewrite the second logarithm over base 2, then combine with a log law.

3441. Consider $\text{f}(x) = -x^3$ as a counterexample to one direction of implication.

3442. Differentiate implicitly to find the first derivative in terms of x and y. Then differentiate again, using the quotient rule, and substitute your expression for $\frac{dy}{dx}$ back in.

3443. Assume, for a contradiction, that $\text{f}(x)-\text{g}(x) = 0$ is a cubic equation. Consider the nature of its roots. Having shown that $\text{f}(x) - \text{g}(x) = 0$ is not cubic, link this back to the leading coefficients.

3444. (a) The outputs are transformed.
   (b) The inputs are transformed.
   (c) Neither of the above happens.

3445. Write the quartic as $(x^2 + x + 8)(ax^2 + bx + c)$, then equate coefficients.

— Alternative Method —

Use polynomial long division.

3446. Friction always acts to oppose (potential) motion. Consider the directions in which the ends of the board would move, were the roofs smooth.

3447. Show that the second derivative is zero at $x = 0$, and changes sign as x passes through 0. For this you'll need to factorise, and use the fact that $2k-1$ is an odd number.

3448. Use double-angle formula. Of the three versions of the $\cos 2x$ formula, choose the one that will deal with the $+1$.

3449. Solve the inequality $X^2 - X < 1$ first.

3450. Assume, for a contradiction, that the polynomial graph $y = \text{f}(x)$ is everywhere convex, and that three points A, B, C on it are collinear. Let their x coordinates be $a < b < c$. Consider the gradient of AB and the gradient of BC.

3451. Rewrite the integrand as a constant plus a proper algebraic fraction.

3452. Rearrange to make the trig functions the subject of each. Then square the equations and add them. The addition of $\frac{\pi}{6}$ doesn't change the final graph, only its parametrisation.

3453. (a) Use trig.
   (b) Differentiate the equation in (a) with respect to time. Your answers should be in rad/s.

3454. Consider replacing x by $x + k$.

3455. Factorise the top as a difference of two squares. Then cancel a common factor and take the limit.

3456. For f to be invertible, it must be one-to-one. This is guaranteed (f having no discontinuities in the domain) if there are no SPs in the domain.

3457. Set up an equation and solve. The equation is a disguised quadratic in e^c.

3458. Factorise the RHS. Then square both sides, noting that, in doing so, you introduce new points.

3459. Consider the possibility space before the black counter is added. Adding the black counter then changes the quantities, but not the probabilities. This gives

$$\mathbb{P}(\text{BBB in the bag}) = \tfrac{1}{4}$$
$$\mathbb{P}(\text{BBW in the bag}) = \tfrac{1}{2}$$
$$\mathbb{P}(\text{BWW in the bag}) = \tfrac{1}{4}.$$

Now, use the conditional information regarding the counter taken out being black.

3460. Use the factor theorem. Since $a = b$ is a root of $a^n - b^n = 0$, $(a - b)$ is a factor.

3461. It intersects one of the three graphs.

3462. Find an expression for U_5 using the standard AP formula. Then consider harmonic form.

3463. Let one projectile have initial speed u at angle θ above the horizontal, and the other speed u in the opposite direction, at θ below the horizontal.

— Alternative Method —

Consider the relative motion of the two parts. In this manner, you can bypass dealing with angles and gravitational acceleration entirely.

3464. Use $y - y_1 = m(x - x_1)$.

3465. Use a prime-factor argument.

3466. The statement is false.

3467. The small triangle is equilateral. Hence, by similar triangles, the larger triangle formed by the points at the circumference is also equilateral.

3468. Substitute the linear equation into the quartic and simplify. Then (using a polynomial solver if you need) factorise the resulting expression. At the end, use the discriminant.

3469. Express the probabilities in terms of n, using the factorial definition of ${}^n\mathrm{C}_r$. Then solve.

3470. Set up an equation for intersections, and look for a double root.

— Alternative Method —

Write down the equations of the tangents to the circle at the y axis, then differentiate the other curve implicitly, and show that $\frac{dy}{dx} = 0$.

3471. (a) Set the x coordinates equal to each other and solve for t. Show that the y coordinates are different at this time.
(b) Find the position of each particle when $y = 2$.

3472. This is a cubic in e^x.

3473. Calculate the area under the cosine wave, then subtract the area of the quarter-circle.

3474. Sketch $y = \text{LHS}$ and $y = \text{RHS}$ on a single set of axes. Then show algebraically that the former is always above the latter.

3475. Square the proposed root and equate coefficients.

3476. (a) The DE is $\frac{dP}{dt} = P(a + bt)$.
(b) You're interested in the rate, as opposed to P itself. So, consider the DE itself, as opposed to its solution. The DE has the rate itself as the subject.
(c) Set $\frac{dP}{dt} = 0$.

3477. Find a counterexample with a non-zero constant of integration.

3478. Differentiate by the product rule to find the first and second derivatives.

3479. The substitution produces a quadratic in z.

3480. Equate (twice) the differences, and so set up an equation in b_3 and b_4. Show that this generalises to the required result.

3481. Sketch the graph $y = |\sin x|$.

3482. Find the equations of the tangent lines, show that they have the same y intercept.

3483. Since the output of the cosine function oscillates between -1 and 1, sketch the graphs $y = \ln(x+1)$ and $y = -\ln(x + 1)$. The curve then oscillates between them.

3484. By symmetry, the centre of the circle must lie on the y axis, below the origin. Set up an equation for the radius r summing y distances to $\frac{4}{3}$.

3485. There's no trick here, you just have to smash out the algebra. Use the quotient rule twice. As ever when algebra gets heavy, make very sure you've simplified as much as possible before proceeding to the next stage.

3486. For a product ab to be negative, precisely one of a and b must be be negative.

3487. Start with the first Pythagorean trig identity, and rearrange to make $\operatorname{cosec}^2\theta$ the subject. Divide top and bottom of the resulting fraction by $\cos^2\theta$.

3488. For a conditioning approach, start from the end. Choose the last letter, then the second last letter, then the rest.

——— Alternative Method ———

For a combinatorial approach, consider all of the letters as distinct, and work with the 8! equally likely outcomes in the possibility space.

3489. Solve $\tan x = \sqrt{3}$ first, then consider what extra possibilities the mod function offers.

3490. Multiply out, and compare the leading coefficient. This tells you that at least one of a, b, c must be equal to 1. So, there is an integer root. Find this, and use it to factorise fully and solve.

3491. (a) Differentiate twice.
(b) Your answer should be in terms of R.
(c) Use your expressions from (a) and (b).
(d) The curves are tangent at the origin.

3492. Consider the degree of the polynomials on the LHS and RHS of the differential equation. Show that these cannot be equal.

3493. Find the x intercepts of the circle. Then work out the angle subtended at the centre by the minor segment, using basic trig.

3494. The student has confused the word "increasing", which refers to $f'(x)$, with the word "convex", which refers to $f''(x)$.

3495. The first task is to sketch the pentagon. Start with the line of symmetry. Place the two sides of length 2 as reflections in the mirror line, with 60° between them. Then consider the possibilities for the sides of length 1. One side must cross the line of symmetry at right angles.

3496. Consider the inputs $x = 0$ and $x = \pi$. You don't want to get into calculus here.

3497. The normals to the two planes are a space diagonal and an edge. Determine the angle between these using the cosine rule.

3498. (a) Find x and y in terms of t, and eliminate t.
(b) Set $y = 0$ and solve. Find the distance between the roots.
(c) The new parabola is a translation of the old one, in the x direction. So, apply an input transformation, i.e. replace x by $x - k$.

3499. (a) Use the quotient rule and a Pythagorean trig identity.
(b) Use $y - y_1 = m(x - x_1)$.
(c) Use either the Newton-Raphson method or fixed-point iteration.

3500. The addition of z coordinates makes no difference here. Set $z = 0$ and solve.

——— End of 35th Hundred ———